

Deacon's Challenge

No 99 - Answer

Stating any assumptions you make calculate the pH of the following solutions:

- a) 0.01 M NaOH
 b) 0.02 M lactic acid
 c) A solution obtained by mixing 10 mL of solution a) with 10 mL of solution b)

(The ionic product of water (K_w) is 1.0×10^{-14} and the dissociation constant of lactic acid (K_a) is 1.38×10^{-4} mol/L.)

- a) By definition $K_w = [\text{H}^+] \times [\text{OH}^-] = 1.0 \times 10^{-14}$

Taking $-\log_{10}$ of all terms gives:

$$-\log_{10} [\text{H}^+] + (-\log_{10} [\text{OH}^-]) = -\log_{10}(1.0 \times 10^{-14})$$

Which simplifies to:

$$\text{pH} + \text{pOH} = 14$$

Assuming complete dissociation of NaOH, $[\text{OH}^-] = 0.01$ mol/L

$$\text{pOH} = -\log_{10} [\text{OH}^-] = -\log_{10} 0.01 = -(-2) = 2$$

$$\text{pH} = 14 - \text{pOH} = 14 - 2 = 12$$

- b) The dissociation of a weak acid (HA) can be written



and its dissociation constant (K_a) is given by:

$$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]}$$

Ignoring the minute contribution to $[\text{H}^+]$ from the dissociation of water, $[\text{A}^-] = [\text{H}^+]$ so that:

$$K_a = \frac{[\text{H}^+]^2}{[\text{HA}]}$$

A further approximation for a weak acid is that only a minute proportion is dissociated so that $[\text{HA}] = [\text{HA}]_{\text{Total}}$ and the value for K_a can be written:

$$K_a = \frac{[\text{H}^+]^2}{[\text{HA}]_{\text{Total}}}$$

which can be rearranged to the following useful expression:

$$[\text{H}^+] = \sqrt{K_a \times [\text{HA}]_{\text{Total}}}$$

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Substituting $[\text{HA}]_{\text{Total}} = [\text{Lactic acid}]_{\text{Total}} = 0.02$ mol/L and $K_a = 1.38 \times 10^{-4}$ mol/L and solving for $[\text{H}^+]$:

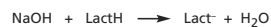
$$[\text{H}^+] = \sqrt{(1.38 \times 10^{-4} \times 0.02)} = \sqrt{(2.76 \times 10^{-6})} = 1.66 \times 10^{-3} \text{ mol/L}$$

$$\text{and } \text{pH} = -\log_{10} [\text{H}^+] = -\log_{10} (1.66 \times 10^{-3}) = -(-2.78) = 2.78$$

(Without assuming that $[\text{Lact}]_{\text{Total}} = [\text{LactH}]$ the solution of the resulting quadratic gives a similar answer of 2.80).

- c) By mixing equal volumes of solutions a) and b) the resulting concentrations are half the initial values i.e. $[\text{NaOH}] = 0.005$ mol/L and $[\text{Lact}]_{\text{Total}} = 0.01$ mol/L.

NaOH neutralizes half of the lactic acid:



so that the new concentrations are:

$$[\text{LactH}] = 0.01 - 0.005 = 0.005 \text{ mol/L}$$

$$[\text{Lact}^-] = 0 + 0.005 = 0.005 \text{ mol/L}$$

By definition when the concentrations of the acid and salt forms of a buffer pair are equal (so that $[\text{salt}]/[\text{acid}] = 1$ and $\log_{10} 1 = 0$) the pH is equal to $\text{p}K_a$.

$$\text{Therefore } \text{pH} = \text{p}K_a = -\log_{10} K_a = -\log_{10} (1.38 \times 10^{-4}) = -(-3.86) = 3.86$$

Question 100

A patient is infused with a drug at the rate of 100 µg/min until a steady state plasma concentration of 100 µg/dL is achieved.

Calculate the clearance of the drug in mL/min.

Comment on your answer.