Deacon's Challenge
No 99 - Answer

Stating any assumptions you make calculate the pH of the following solutions:

a) 0.01 M NaOH
b) 0.02 M lactic acid
c) A solution obtained by mixing 10 mL of solution a) with 10 mL of solution b)

(The ionic product of water (K_w) is 1.0 x 10^{-14} and the dissociation constant of lactic acid (K_a) is 1.38 x 10^{-4} mol/L.)

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a) By definition $K_w = [H^+] \times [OH^-] = 1.0 \times 10^{-14}$

Taking $-\log_{10}$ of all terms gives:

$$-\log_{10}[H^+] + (-\log_{10}[OH^-]) = -\log_{10}(1.0 \times 10^{-14})$$

Which simplifies to:

$$\boxed{pH + pOH = 14}$$

Assuming complete dissociation of NaOH, $[OH^-] = 0.01$ mol/L

$$pOH = -\log_{10}[OH^-] = -\log_{10} 0.01 = -(-2) = \mathbf{2}$$

$$pH = 14 - pOH = 14 - 2 = 12$$

b) The dissociation of a weak acid (HA) can be written

$$HA \longleftrightarrow H^+ + A^-$$

and its dissociation constant (K_a) is given by:

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

Ignoring the minute contribution to $[H^+]$ from the dissociation of water, $[A^-] = [H^+]$ so that:

$$K_a = \frac{[H^+]^2}{[HA]}$$

A further approximation for a weak acid is that only a minute proportion is dissociated so that $[HA] = [HA]_{Total}$ and the value for K_a can be written:

$$K_a = \frac{[H^+]^2}{[HA]_{Total}}$$

which can be rearranged to the following useful expression:

$$\boxed{[H^+] = \sqrt{(K_a \times [HA]_{Total})}}$$

Substituting $[HA]_{Total}$ = $[\text{Lactic acid}]_{Total}$ = 0.02 mol/L and $K_a = 1.38 \times 10^{-4}$ mol/L and solving for $[H^+]$:

$$[H^+] = \sqrt{(1.38 \times 10^{-4} \times 0.02)} = \sqrt{(2.76 \times 10^{-6})} = 1.66 \times 10^{-3} \text{ mol/L}$$

and $pH = -\log_{10}[H^+] = -\log_{10}(1.66 \times 10^{-3}) = -(-2.78) = \mathbf{2.78}$

(Without assuming that $[Lact]_{Total}$ = [LactH] the solution of the resulting quadratic gives a similar answer of 2.80).

c) By mixing equal volumes of solutions a) and b) the resulting concentrations are half the initial values i.e. [NaOH] = 0.005 mol/L and $[Lact]_{Total}$ = 0.01 mol/L.

NaOH neutralizes half of the lactic acid :

$$NaOH + LactH \longrightarrow Lact^- + H_2O$$

so that the new concentrations are:

$$[LactH] = 0.01 - 0.005 = 0.005 \text{ mol/L}$$

$$[Lact^-] = 0 + 0.005 = 0.005 \text{ mol/L}$$

By definition when the concentrations of the acid and salt forms of a buffer pair are equal (so that [salt]/[acid] = 1 and $\log_{10}1 = 0$) the pH is equal to p*Ka*.

Therefore $pH = pKa = -\log_{10} Ka = -\log_{10}(1.38 \times 10^{-4}) = -(-3.86) = \mathbf{3.86}$

Question 100

A patient is infused with a drug at the rate of 100 µg/min until a steady state plasma concentration of 100 µg/dL is achieved.

Calculate the clearance of the drug in mL/min.

Comment on your answer.