Deacon's Challenge

No 182 - Answer

Your Consultant Endocrinologist has expressed concern that two blood glucose monitors on his ward are yielding discrepant results. As part of your investigation you perform replicate measurements on a QC material on both instruments with the following results:

	Number of results (n)	*Mean* (m)	*Standard deviation* (s)
Instrument 1	*5*	*5.6*	*0.12*
Instrument 2	*7*	*6.0*	*0.14*

Does this data support his suspicion?

Two tailed t-distribution:

Degrees of freedom	P			
	0.10	*0.05*	*0.02*	*0.01*
9	*1.833*	*2.262*	*2.821*	*3.250*
10	*1.812*	*2.228*	*2.764*	*3.169*
11	*1.796*	*2.201*	*2.718*	*3.106*
12	*1.782*	*2.179*	*2.681*	*3.055*
13	*1.771*	*2.160*	*2.650*	*3.012*

The obvious approach is to compare the two means using a *t*-test. The principal is that the difference between the two means will form a Gaussian distribution and if this difference is insignificant then their difference will not differ from an overall mean of zero (the *null hypothesis*). When dealing with mean values their variation (i.e. standard deviation of the sampling distribution of the mean) is described by the standard error (*SE*) which is dependent on the sample size (*n*) used to calculate the mean:

$$SE = s/\sqrt{n}$$

The value for *t* will be given by the difference between the two means ($m_1 - m_2$) divided by the standard error of the differences ($SE_{difference}$):

$$t = \frac{m_1 - m_2}{SE_{difference}}$$

$SE_{difference}$ will be a combination of the individual standard errors of the two means. The way in which this is done is controversial and the approach used depends on whether the standard deviations of the two means are significantly different. This could be formally evaluated using a variance ratio test but the difference is unlikely to be significant if the ratio of the larger to the smaller value is less than 2.

Whenever two results are added (or subtracted) the standard deviation of the result is the square root of the sum of squares of their individual standard deviations. The same applies to standard errors:

$$SE_{difference} = \sqrt{(SE_1^2 + SE_2^2)} = \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$$

N.B. the square of $s/\sqrt{n}$ (i.e. $s/\sqrt{n} \times s/\sqrt{n}$) is s^2n.

So that the expression for *t* becomes:

$$t = \frac{m_1 - m_2}{\sqrt{\{(s_1^2/n_1) + (s_2^2/n_2)\}}}$$

with $n_1 + n_2 - 2$ degrees of freedom

Substituting into this expression:

$$t = \frac{5.6 - 6.0}{\sqrt{\{(0.12^2/5) + (0.14^2/7)\}}} = \frac{-0.4}{\sqrt{(0.00288 + 0.00280)}}$$

$$= \frac{-0.4}{\sqrt{0.00568}} = \frac{-0.4}{0.0754} = -5.31 \text{ (the negative sign can be ignored)}$$

The degrees of freedom (*d.f.*) $= n_1 + n_2 - 2 = 5 + 7 - 2 = 10$

From tables the probability (*P* value) of obtaining a t value greater than 5.31 by pure chance is less than 0.01. Therefore there is a significant difference (taking a P value of 0.05 as a decision level) in the mean values obtained with the two instruments. Whether or not this difference actually matters cannot be determined from statistical tests!

Question 183

A medical team is considering a change of screening strategy for a disease X. The traditional test A has a sensitivity of 95% and specificity of 60% for disease X. A new test B has been introduced which has an increased specificity 75% for this disease but a reduced sensitivity of 75%. Calculate the positive and negative predictive values for each test for a population in which X has a true prevalence of 10%.

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