Deacon's Challenge

No 191 - Answer

You need to make up a phosphate buffer with a pH of 7.4 and a total phosphate concentration of 40 mmol/L. Calculate the amounts of sodium dihydrogen phosphate and disodium monohydrogen phosphate that need to be weighed into 1 litre of water, given that the pKa is 6.82 (atomic weights: Na 23, P 31).

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The Henderson-Hasselbalch equation relates the concentrations of acid and salt to pH:

$$\text{pH} = \text{p}K\text{a} + \log_{10}\frac{[\text{salt}]}{[\text{acid}]}$$

The dihydrogen phosphate ion dissociates to give monohydrogen phosphate and hydrogen ions:

$$H_2PO_4^- \longrightarrow HPO_4^{2-} + H^+$$

Therefore the relevant form of the Henderson-Hasselbalch equation is:

$$\text{pH} = \text{p}K\text{a} + \log_{10}\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$$

Only the total phosphate concentration is given so express the concentration of one in terms of the other:

$$\text{Total phosphate} = [HPO_4^{2-}] + [H_2PO_4^-] = 40 \text{ mmol/L}$$

Therefore $[HPO_4^{2-}] = 40 - [H_2PO_4^-]$

Substitute this, pH = 7.4 and p*K*a = 6.82 into the Henderson-Hasselbalch equation and solve for $[H_2PO_4^-]$:

$$7.4 = 6.82 + \log_{10}\frac{(40 - [H_2PO_4^-])}{[H_2PO4\text{-}]}$$

$$\log_{10}\frac{(40 - [H_2PO_4^-])}{[H_2PO_4^-]} = 7.4 - 6.82 = 0.58$$

$$\frac{(40 - [H_2PO_4^-])}{[H_2PO_4^-]} = \text{antilog}_{10}\, 0.58 = 3.80 \qquad \text{(to 3 sig figs)}$$

$$40 - [H_2PO_4^-] = 3.80\,[H_2PO_4^-]$$

$$40 = 3.80[H_2PO_4^-] + [H_2PO_4^-] = 4.80[H_2PO_4^-]$$

$$[H_2PO_4^-] = \frac{40}{4.80} = 8.33 \text{ mmol/L}$$

The concentration of the other species [HPO_4^{2-}] is calculated by difference:

$$[HPO_4^{2-}] = 40 - 8.33 = 31.67 \text{ mmol/L}$$

For each salt:

$$\text{Conc}^n \text{ (g/L)} = \text{Conc}^n \text{ (mol/L)} \times \text{MW} = \frac{\text{Conc}^n \text{ (mmol/L)}}{1{,}000} \times \text{MW}$$

$$\text{MW } NaH_2PO_4 = 23 + (2 \times 1) + 31 + (4 \times 16) = 120$$

$$\text{MW } Na_2HPO_4 = (2 \times 23) + 1 + 31 + (4 \times 16) = 142$$

$$\text{Wt } NaH_2PO_4 \text{ to make 1L} = \frac{8.33}{1{,}000} \times 120 = \mathbf{1.00\ g} \quad \text{(to 3 sig figs)}$$

$$\text{Wt } Na_2HPO_4 \text{ to make 1L} = \frac{31.67}{1{,}000} \times 142 = \mathbf{4.50\ g} \quad \text{(to 3 sig figs)}$$

Question 192

A metabolic disease is known to result in decreased plasma activity of enzyme X. X was measured in 100 normal subjects and 100 individuals with the disease. A reasonable Gaussian distribution was obtained for each population with the following statistics:

	Mean (m)	Standard deviation (s)
Normal subjects	1025 U/L	100 U/L
Diseased group	530 U/L	200 U/L

Find the decision level at which sensitivity is equal to specificity. What is the sensitivity (and hence specificity) at this decision level?

Two-tailed values of the normal deviate (z-score) and probability (P) are:

P(%)	10	5	2	1	0.2	0.1
z	1.65	1.96	2.33	2.58	3.09	3.29