Deacon's Challenge

No 151 - Answer

As part of the evaluation of a new serum creatinine assay a quality control sample is analysed in duplicate on twenty consecutive days with the following results (µmol creatinine/L serum):

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1st result	100	98	101	99	104	96	98	100	101	99	103	101	99	94	100	98	95	100	100	101
2nd result	102	96	104	101	101	94	101	104	96	97	104	99	102	97	101	96	94	101	99	103

Calculate the within-day imprecision.

The variance (s^2) of n values for x with mean m is given by the formula $\Sigma(x - m)^2/(n^{-1})$. It would be a simple matter to calculate the within-day precision from any of the days upon which the measurements were taken. However, a more reliable estimate would be obtained if the data could be pooled from all 20 days, especially as the within-day imprecision may vary from day-to-day. However, if the above formula was applied to all 40 results using the overall mean then the result would be the *total* analytical variance which is made up of two components – the *within-day* and *between-day* variance. These two components can be separated if the appropriate mean values are used in the calculation (a classical two way analysis of variance).

The variance on each day is independent of the between-day variance. Therefore the individual variances (each calculated from the two individual results and their *daily* mean) could be combined to give a better estimate than would be obtained for any single pair of results:

For day 1 there are two results (100 and 102) with a mean of 101 so the calculation becomes:

$$\Sigma(x - m)^2/(n^{-1}) = [(100 - 101)^2 + (102 - 101)^2]/(2-1) = (-1^2 + 1^2)/1 = 1 + 1 = 2$$

and for day 2 (x = 98 and 96 with m = 97):

$$\Sigma(x - m)^2/(n^{-1}) = [(98 - 97)^2 + (96 - 97)^2]/(2-1) = 2$$

This process could be continued for each day, all the daily variances added together then divided by 20 to give the average daily variance. However, since for each day the mean values cancel it is possible to take a short cut by calculating the difference, d, between each pair of duplicates. A useful property of duplicates is that the mean subtracted from either result is always $d/2$ (or $-d/2$) so that on any day:

$$\frac{\Sigma(x - m)^2}{n - 1} = \frac{(d/2)^2 + (d/2)^2}{2 - 1} = \frac{d^2/4 + d^2/4}{1} = \frac{2\,d^2}{4} = \frac{d^2}{2}$$

If all the values for d^2 are added together then divided by the number of days (n) then the following simplified formula can be used to calculate the within-day variance:

$$s^2_{\text{Within-day}} = \frac{\Sigma d^2}{2n}$$

Applying this formula to the above data:

$$s^2_{\text{Within-day}} = \frac{d_1^2 + d_2^2 + d_3^2 + \ldots\ldots d_{20}^2}{2 \times 20}$$

$$= \frac{(100 - 102)^2 + (98 - 96)^2 + (101 - 104)^2 + \ldots\ldots(101 - 103)^2}{40}$$

$$= \frac{123}{40} = \mathbf{3.08\ (\mu mol/L)^2}\ \text{(to 3 sig figs)}$$

The standard deviation is simply the square root of the variance:

$$s_{\text{Within-run}} = \sqrt{3.08} = \mathbf{1.75\ \mu mol/L}$$

Expression of the standard deviation as a percentage of the overall mean (99.5 µmol/L) gives the coefficient of variation:

$$cv_{\text{Within-run}} = \frac{1.75 \times 100}{99.5} = \mathbf{1.8\%}\ \text{(to 2 sig figs)}$$

Question 152

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1st result	100	98	101	99	104	96	98	100	101	99	103	101	99	94	100	98	95	100	100	101
2nd result	102	96	104	101	101	94	101	104	96	97	104	99	102	97	101	96	94	101	99	103

Calculate the between-day imprecision.