Deacon's Challenge No. 53 Answer

A woman had a beta hCG concentration measured at 265 IU/L and 11 days later, following some abdominal pain, it was 820 IU/L. Assuming hCG rises exponentially in early pregnancy, what has been the doubling time over this period? What is the significance of the result you obtain?

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The integrated form of the rate equation describing exponential growth is:

$$\log_e C_t = \log_e C_0 + k.t$$

Where: C_t = concentration at time t
C_0 = initial concentration
t = time
k = rate constant

Note that this formula is identical to that used for exponential decay except the $-k.t$ has become $+k.t$.

Substitute: C_t = hCG concentration at 11 days = 820 IU/L
C_0 = initial hCG concentration = 265 IU/L
t = 11 days

Then solve for k:

$$\log_e 820 = \log_e 265 + 11\,k$$
$$6.709 = 5.580 + 11\,k$$
$$11\,k = 6.709 - 5.580 = 1.129$$
$$k = \frac{1.129}{11} = 0.103 \text{ days}^{-1}$$

We are asked to calculate the doubling time (t_2). When $t = t_2$ the initial concentration of hCG has doubled to $2C_0$. Therefore

$$\log_e 2C_0 = \log_e C_0 + k.t_2$$

Rearrange to give an expression for t_2:

$$k.t_2 = \log_e 2C_0 - \log_e C_0 = \log_e \frac{(2C_0)}{C_0} = \log_e 2 = 0.693$$

$$t_2 = \frac{0.693}{k}$$

Substitute k = 0.103 to obtain the doubling time for hCG:

$$\text{Doubling time } (t_2) = \frac{0.693}{0.103} = 6.7 \text{ days (2 sig figs)}$$

The normal doubling time for hCG during early pregnancy is approximately 2 days. Therefore this result is consistent with ectopic pregnancy.

Alternatively the value for k ($0.693/t_2$) can be substituted directly into the integrated rate equation and re-arranged to give an expression for t_2:

$$\log_e C_t = \log_e C_0 + \frac{0.693.t}{t_2}$$

$$t_2 = \frac{0.693.t}{(\log_e C_t - \log_e C_0)}$$

Question 54

An estimation of glomerular filtration rate can be calculated using the abbreviated MDRD (Modified Diet in Renal Disease) formula:

GFR (mL/min/1.73m^2) = 186 x [serum creatinine/88.4]$^{-1.154}$ x [age in years]$^{-0.203}$

x 0.742 if female and/or x 1.21 if Afro American origin (where serum creatinine is in µmol/L).

Calculate the GFR for a 55 year old Caucasian women whose serum creatinine is 125 µmol/L, and her creatinine clearance, given that a 24h urine collection with a volume of 1.1 L had a creatinine concentration of 4.7 mmol/L.

Comment critically on the two values.

MRCPath, May 2005