Deacon's Challenge No. 34 Answer

Christmas Special

What is the pH of a 1.0 x 10^{-8} molar solution of hydrochloric acid?

$$pH = \log_{10} \frac{1}{[H^+]}$$

If it is assumed that the hydrochloric acid is completely dissociated

$$HCl \longrightarrow H^+ + Cl^-$$

and that hydrogen ions only arise from hydrochloric acid (i.e. that $[H^+] = 1.0 \times 10^{-8}$ M), then:

$$pH = \log_{10} \frac{1}{1.0 \times 10^{-8}} = \log_{10} 1.0 \times 10^{8} = 8$$

Clearly this is nonsense because a solution of acid can never be alkaline!

At very low concentrations it is also important to consider the dissociation of water which itself contributes to the hydrogen ion concentration:

$$H_2O \longrightarrow H^+ + OH^-$$

So that $[H^+]_{Total} = [H^+]_{From\ HCl} + [H^+]_{From\ water}$

Usually the hydrogen ion contribution from water is small in comparison to the hydrogen ions derived from added acid and is ignored (textbooks never seem to point this out). At neutral pH the hydrogen ion concentration of pure water is 1.0 x 10^{-7} mol/L. One approach is to assume that this is the contribution from water for a very dilute acid which must have a pH near to neutrality:

$$[H^+]_{Total} = (1.0 \times 10^{-8}) + (1.0 \times 10^{-7}) = 1.1 \times 10^{-7} \text{ mol/L}$$

and $$pH = \log_{10} \frac{1}{1.1 \times 10^{-7}} = \log_{10} 9.09 \times 10^{6} = 6.96$$

Strictly speaking the hydrogen ions from HCl will cause minor suppression of the dissociation of water in order to keep the ionic product of water (K_w) constant:

$$K_w = [H^+] \times [OH^-] = 10^{-14}$$

Since one molecule of water always yields one hydrogen ion and one hydroxide ion, then the expression for the ionic product of water can also be written:

$$K_w = \{ [H^+]_{from\ HCl} + [OH^-] \} \times [OH^-]$$

$$1.0 \times 10^{-14} = \{ (1.0 \times 10^{-8}) + [OH^-] \} \times [OH^-]$$

$$1.0 \times 10^{-14} = (1.0 \times 10^{-8}) [OH^-] + [OH-]^2$$

Which is a quadratic equation which can be rearranged to:

$$[OH^-]^2 + (1.0 \times 10^{-8}) [OH^-] - 1.0 \times 10^{-14} = 0$$

and solved in the usual way:

$$[OH^-] = \frac{- (1 \times 10^{-8}) \pm \sqrt{ \{ (1 \times 10^{-8})2- [4 \times 1 \times (1 \times 10^{-14})] \} }}{2 \times 1}$$

$$= 0.995 \times 10^{-7} \text{ mmol/L}$$

Using this value as the hydrogen ion component from water the pH can again be calculated:

$$[H^+]^{Total} = (0.995 \times 10^{-7}) + (1.0 \times 10^{-8}) = 1.095 \times 10^{-7} \text{ mmol/L}$$

$$pH = \log_{10} \frac{1}{1.095 \times 10^{-7}} = \log_{10} 9.132 \times 10^{-6} = 6.96 \text{ (3 sig figs)}$$

Which is the same result (to 3 significant figures) obtained if the suppression of ionization of water by the hydrogen ions derived from hydrochloric acid is ignored.

Question No. 35

A 60 mg dose of a drug is given to a male experimental subject who weighs 80 kg. Assuming that the drug is completely absorbed and distributed evenly throughout the total body water, estimate the potential peak plasma level. If the drug were distributed only within the extracellular compartment, what would the plasma level be?

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