Deacon's Challenge

No 96 - Answer

A patient who is known to have diabetes insipidus is admitted in a semicomatose state. His serum sodium concentration is 155 mmol/L. His admission weight is 79 Kg. Estimate his water deficit, indicating clearly any assumptions you make.

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The water deficit is the difference between the total body water at presentation (the final value) and an initial value some time in the past when the patient was fully hydrated. If we assume that any difference in body weight over this unknown time period is due to water loss alone, then we can write:

$$\text{Initial body weight (Kg)} = \text{Final body weight (Kg)} + \text{Water deficit (Kg)}$$

Substituting the final body weight of 79 Kg:

$$\text{Initial body weight (Kg)} = 79 + \text{Water deficit}$$

Assuming that initially the patient had a normal body water content for an adult male of 60% (i.e. was fully hydrated and not obese) then the initial body water is given by:

$$\text{Initial body water (L)} = (79 + \text{Water deficit}) \times 0.6$$
$$= 47.4 + (0.6 \times \text{Water deficit})$$

Subtraction of the water deficit from this value gives the final body water volume:

$$\text{Final body water (L)} = 47.4 + (0.6 \times \text{Water deficit}) - \text{Water deficit}$$
$$= 47.4 - (0.4 \times \text{Water deficit})$$

Diabetes insipidus is due to a lack of ADH which reduces renal tubular water reabsorption leading to inappropriate water loss. Water loss from the plasma (and hence ECF) results in hyperosmolality which stimulates water shift from the ICF to ECF in order to maintain osmotic equilibrium. Therefore water loss is shared between both compartments, each of which becomes equally hyperosmolar. The major determinant of ECF osmolality is the plasma sodium concentration. The ICF has a different solute composition but since its osmolality must be equal to that of the ECF we can treat it, for the purposes of this calculation, as if an equivalent sodium concentration also determines ICF osmolality. Therefore we can write the following expression for plasma sodium concentration:

$$\text{Plasma sodium concentration (mmol/L)} = \frac{\text{Total body sodium (mmol)}}{\text{Total body water (L)}}$$

which can be rearranged to:

$$\text{Total body sodium (mmol)} = \text{Plasma sodium concentration (mmol/L)} \times \text{Total body water (L)}$$

Assuming that sodium has been conserved (i.e. pure water loss has occurred) and there has been no significant change in other solutes, then the relationship between initial and final values for plasma sodium concentration and body water volumes is:

$$\text{Initial sodium (mmol/L)} \times \text{Initial water (L)} = \text{Final sodium (mmol/L)} \times \text{Final water (L)}$$

Assuming initially a normal plasma sodium (140 mmol/L) then subsitituing this value, the final plasma sodium (155 mmol/L) and the expressions for initial and final body water volumes gives:

$$140 \times \{47.4 + (0.6 \times \text{Water deficit})\} = 155 \times \{47.4 - (0.4 \times \text{Water deficit})\}$$

which can then be solved for water deficit:

$$47.4 + (0.6 \times \text{Water deficit}) = \frac{155 \times \{[47.4 - (0.4 \times \text{Water deficit})\}}{140}$$

$$47.4 + (0.6 \times \text{Water deficit}) = 52.5 - (0.44 \times \text{water deficit})$$

$$(0.6 \times \text{water deficit}) + (0.44 \times \text{water deficit}) = 52.5 - 47.4$$

$$1.04 \times \text{Water deficit} = 5.1$$

$$\text{Water deficit} = \frac{5.1}{1.04} = \mathbf{4.9\ L}$$

This calculation could be considerably simplified by making the further assumption that initially the patient had a normal body water content for an adult male of 42 L:

$$\text{Final water volume} = \frac{140 \times 42}{155} = 37.9\ \text{L} \quad \text{and water deficit} = 42 - 37.9 = 4.1\ \text{L}$$

Since we have to make a number of other assumptions anyway, and the final result is only used as an approximate guide to fluid replacement, it could be argued that, for practical purposes, this assumption is quite reasonable.

Question 97

A 14-year old boy was admitted in an intoxicated state having been suspected to have consumed an unknown quantity of methylated spirit (a mixture of ethanol and methanol). He weighed 55 Kg. The following laboratory results were obtained on a blood sample taken within 15 minutes of admission:

Urea	2.9 mmol/L
Sodium	136 mmol/L
Potassium	4.2 mmol/L
Glucose	3.2 mmol/L
Osmolality	325 mOsm/Kg
Ethanol	185 mg/dL

a) Estimate his serum methanol concentration in mg/dL and the volume of methanol he consumed (volume of distribution 0.6 L/Kg, density 0.791 g/mL) stating any assumptions you make or limitations of this approach.

b) Why is the toxicity associated with this methanol dose likely to be less than if he had consumed an equivalent volume of pure methanol?

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