Deacon's Challenge

No 189 - Answer

A 44-year old woman, who suffered a miscarriage of pregnancy four days previously, was found to have a serum β-HCG concentration of 658 IU/L. Given that the half-life of β-HCG at more than 48 hours after termination of pregnancy is 56 hours, in how many days time would you expect her serum β-HCG concentration to reach a level of 5 IU/L?

FRCPath, Autumn 2015

Since the miscarriage occurred more than 2 days ago it is appropriate to use the first order rate equation with a half-life of 56 h.

Method 1

Use the integrated form of the 1st order rate equation:

$$\ln Cp_t = \ln Cp_0 - k_d t$$

where Cp_t = final β-HCG = 5 IU/L

Cp_0 = initial β-HCG = 658 IU/L

t = time (in h) taken to reach final β-HCG level = ?

k_d = elimination rate constant = $0.693/t_{1/2}$ = 0.693/56 = 0.01238 h^{-1}

Substitute these values and solve for t:

$$\ln 5 = \ln 658 - 0.01238t$$

$$1.609 = 6.489 - 0.01238t$$

$$0.01238t = 6.489 - 1.609 = 4.88$$

$$t = \frac{4.88}{0.01238} = 394 \text{ h}$$

Divide by 24 to convert hours to days:

$$\text{Days taken} = \frac{394}{24} = \textbf{16.4 days} \quad \textbf{(16 days to 2 sig figs)}$$

Method 2

Use the equation relating concentration ratio to number of half-lives:

$$N = 3.32 \log_{10} CR$$

where: CR is the ratio of concentration at zero time to time t

N = number of half-lives

$$N = 3.32 \log_{10} \frac{658}{5}$$

$$N = 3.32 \log_{10} 132$$

$$= 3.32 \times 2.12$$

$$= 7.04$$

Therefore $t = 7.04 \times t_{1/2} = 7.04 \times 56 =$ **394 h (approx. 16 days)**

NB: Derivation of $N = 3.32 \log_{10} CR$:

Logarithmic form of integrated 1st order rate equation:

$$\ln Cp_t = \ln Cp_0 - k_d t$$

Rearrange:

$$k_d t = \ln Cp_0 - \ln Cp_t$$

$\ln Cp_0 - \ln Cp_t$ can be written $\ln (Cp_0/Cp_t)$

$$k_d t = \ln (Cp_0/Cp_t)$$

Let $CR = Cp_0/Cp_t$, and use $0.693/t_{1/2}$ as k_d:

$$\frac{0.693\, t}{t_{1/2}} = \ln CR$$

Express t in terms of number of half-lives (i.e. $N = t/t_{1/2}$)

$$0.693\, N = \ln CR$$

$$N = \frac{1}{0.693} \ln CR$$

Convert $\ln CR$ to $2.303 \log_{10} CR$

$$N = \frac{2.303 \log_{10} CR}{0.693}$$

$$\boxed{N = 3.32 \log_{10} CR}$$

To do the calculation in one step:

$$\boxed{t = 3.32\, t_{1/2} \log_{10} CR}$$

Question 190

Recent European guidelines advise treatment with urea as a second-line treatment for the syndrome of inappropriate antidiuresis (SIADH), with the aim of inducing an osmotic diuresis. A 65-kg man with a stable serum sodium concentration (after fluid restriction) of 125 mmol/L is commenced on urea 15 g/day. Calculate the anticipated increase in serum osmolality, stating the assumptions made.

NB: the formula of urea is $CO(NH_2)_2$.

FRCPath, Autumn 2015